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22C:019 Homework 4

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4. a. P(1) = 1^3 = [(1(n+1))/2]^2

b. base case: 1^3 = [(1(1+1))/2]^2 = 1

c. inductive hypothesis P(k) is true

1^3 + 2^3… + k^3 = [(k(k+1))/2]^2

d. P(k) implies P(k+1)

1^3 + 2^3… + k^3 + (k+1)^3 = [(k+1(k+2))/2]^2

e. [(k(k+1))/2]^2 + (k+1) = [(k+1(k+2))/2]^2

6. base case : P(1) is true 1x1! = (1+1)!-1

= 1

inductive step: p(k) is true

1x1! + 2x2!+….+kxk! = (k+1)!-1

show P(k+1) is true

1x1! + 2x2!+….+kxk! + (k+1)(k+1)! = (k+1)!-1+(k+1)(k+1)!

= (k+2)!-1

P(k+1) is true

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16. P(n) = 1x2x3+2x3x4 + n(n+1)(n+2) = (n(n+1)(n+2)(n+3))/4

base case: P(1) is true 1x2x3 = (1(1+1)(1+2)(1+3))/4

inductive step : P(k) is true

1x2x3+2x3x4 + k(k+1)(k+2) = (k(k+1)(k+2)(k+3))/4

p(k+1) is true

1x2x3+2x3x4 + k(k+1)(k+2)+ (k+1)(k+2)(k+3)

=((k+1)(k+2)(k+3)(k+4))/4

20. P(n) = 3^n < n for n is an int greater than 6

base step P(7)

3^7 = 2187 and 7! = 5040; 3^< 7!

inductive step: p(k) is ture for k>6

3^k < k! prove p(k+1) is true

P(k+1) = 3^(k+1)

= 3(3^k) <3(k!) <(k+1)!

32. 3 / n^3+2n when n is positive

base case: P(1) = 1^3+2(1) = 3

inductive step: assume p(k) is true

k^3+2k is divisible by 3

p(k+1) is true

(k+1)^3 + 2(k+1) = k^3+1^3+3k^2+3k+2k+2

=(k^3+2K) +3(k^2+k+1) is / by 3

therefore p(k+1) is true

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4.a. base case:

P(18) = one 4 cent stamp and two 7 cent stamps

P(19) = three 4 cent stamps and one 7 cent stamp

P(20) = five 4 cent stamps

P(21) = three 7 cent stamps

b.we can form m stamps for all 18<= j <=k where k>= 21

c. k+1 cent stamps using just 4 cent and 7 cent stamps

d. P(k-3) is true; k-3 stamps. Put one more 4 cent stamp on the envelope and formed k+1 cent postage

e. principle of strong induction is true whenever n>= 18

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2b. f(n+1) = 3f(n)

f(1) = 3f(1) = 3x1 = 3

f(2) = 3f(2) = 3x3 = 9

f(3) = 3f(3) = 3x9 = 27

f(4) = 3f(4) = 3x27 = 81

2d. f(n+1) = f(n)^2 + f(n) + 1

f(1) = f(0)^2 + f(0) + 1 = 1+1+1 = 3

f(2) = f(1)^2 + f(1) + 1 = 9 + 3 + 1 = 13

f(3) = f(2)^2 + f(2) + 1 = 169 + 13 + 1 = 183

f(4) = f(3)^2 + f(3) + 1 = 33489 + 183 + 1 = 33673

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8a. a\_n = 4n-2

a\_1 = 4(1) – 2 = 2

a\_(n+1) = 4(n+1) -2 = 4n-2+4 = 4\_n+4

8d. a\_n = 1+(-1)^n

(-1)^n = a\_n-1

a\_1 = 1+(-1) = 0

a\_n+1 = 1+ (-1)^n+1

= 2-a\_n

a\_1 = 0 and a\_n+1 = 2-a\_n

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4. 72 types

8. 156000 ways